Lec. 4-5

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Maxwell’s Equation

- Maxwell's equations are a set of coupled partial differential equations that, together with the Lorentz force law, form the foundation of classical electromagnetism, classical optics, and electric circuits.
- Maxwell’s equations, four equations that, together, form a complete description of the production and interrelation of electric and magnetic fields.

Equations in Modern Vector Form

Vector is **a quantity that has both magnitude and direction.** The equations given below are represented into vector form

$\nabla \cdot \mathbf{D} = \rho$	(1)	Gauss' Law
$\nabla \cdot \mathbf{B} = 0$	(2)	Gauss' Law for magnetism
$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	(3)	Faraday's Law
$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$	(4)	Ampère-Maxwell Law

Formulation in integral form

Integral equations are **equations in which an unknown function appears under an integral sign**.

Maxwell's Equations in a Medium

Equations	Integral Form	Remark
Gauss' Law for Electricity	$\oiint_A \mathbf{D}\cdot d\mathbf{A} = \iiint_V \rho\, dV$	$\rho = \rho_f$
Gauss' Law for Magnetism	$\oiint_A \mathbf{B}\cdot d\mathbf{A} = 0$	No magnetic monopole
Faraday's Law of Induction	$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\iint_A \mathbf{B}\cdot d\mathbf{A}$	Changing magnetic field produces electric field
Ampere's Law	$\oint_C \mathbf{H}\cdot d\mathbf{l} = \iint_A \left[\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}\right]\cdot d\mathbf{A}$	Current and changing electric field produce magnetic field

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- The statements of these four equations are, respectively:
- Electric field diverges from electric charge, an expression of the Coulomb force
- There are no isolated magnetic poles, but the Coulomb force acts between the poles of a magnet
- Electric fields are produced by changing magnetic fields, an expression of Faraday’s law of induction
- Circulating magnetic fields are produced by changing electric fields and by electric currents

1.Gauss law In Electrostatics:

Gauss law describes the nature of the electric field around electric charges. The law is expressed in terms of electric charge density and electric charge density. Gauss Law The inverted triangle is called the divergence operator.

ρ

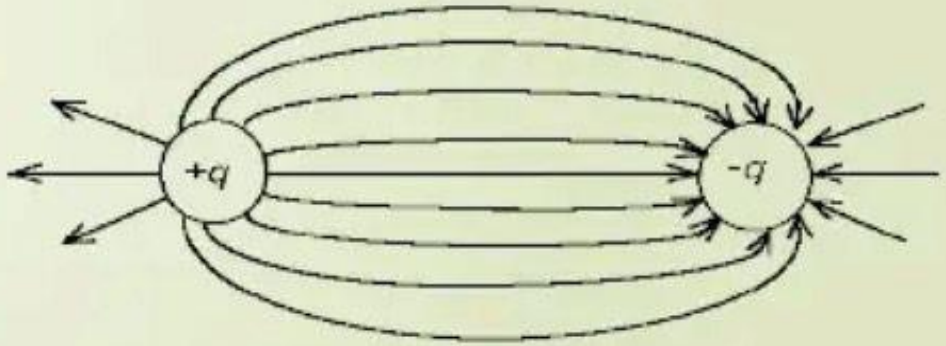
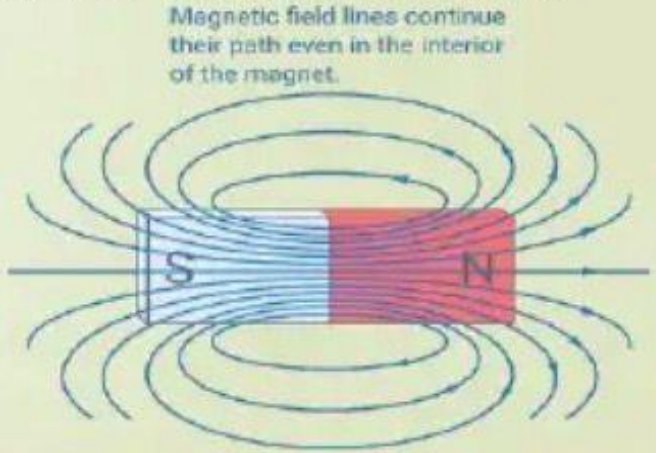


Figure 1.1

- The equations hold good at any point in space. When the electric charge exists any somewhere, the divergence of D at that particular point is nonzero, else it is zero.

2. Gauss's Magnetism Law

- You need to be familiar with Gauss Law for the electric field to understand this equation.



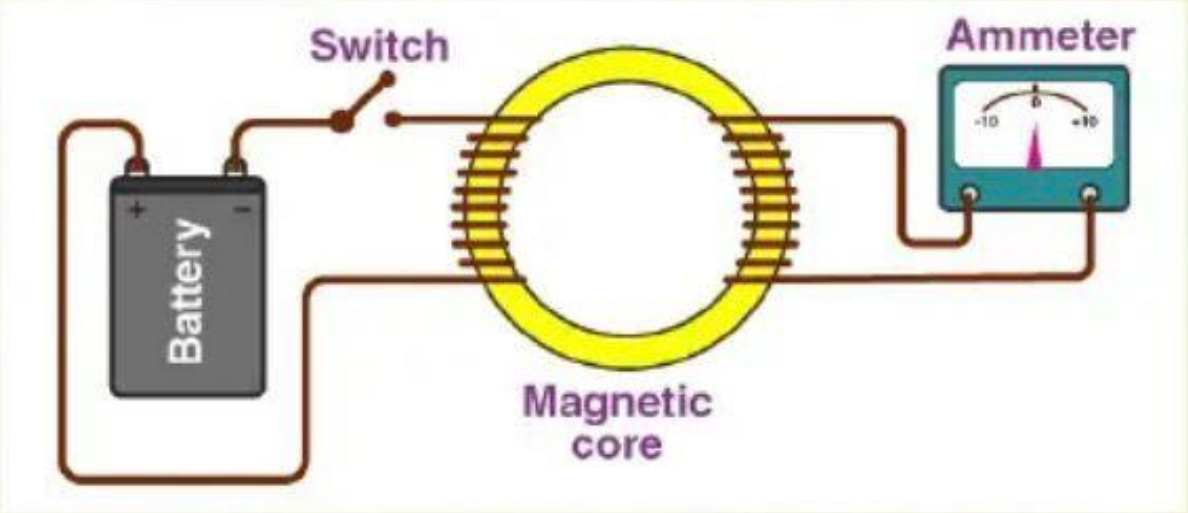
$$\nabla . B = 0$$

- You can see that both the equations indicate the divergence of the field. The top equation states that the divergence of the electric flux density D equals the volume of electric charge density. The second equation states the divergence of the Magnetic Flux Density (B) is null.

3. Faraday's Law

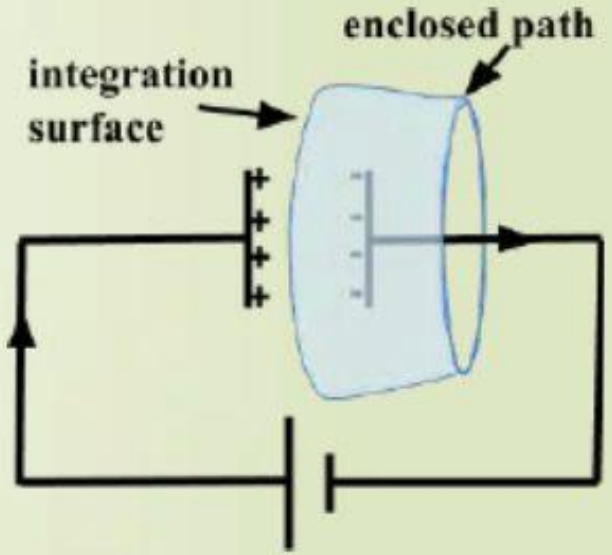
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday was a scientist whose experiment setup led to Faraday's Law which is shown in the figure below.



4. Ampere's Law

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$



The law shows the relationship between the flow of electric current and the magnetic field around it. Suppose the wire carries a current I, the current produces a magnetic field that surrounds the wire.

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The experiment is not very complex. When a battery is disconnected, no electricity flows through the wire. Hence, no magnetic flux is induced in the iron (Magnetic Core). The iron acts like a magnetic field that flows easily in a magnetic material. The purpose of the core is to form a path for the flow of magnetic flux.

Significance of Maxwell Equations

- Maxwell’s synthesis of Electromagnetism in these four equations is one of the greatest milestones of theoretical physics, in compared with Newton’s laws of motion in mechanics. The physical significance of these equations is that each of them represents generalization of certain experimental observations and results, which can be summarized in the following points

- Maxwell's Equations: The Foundation of Electromagnetism
- These four equations describe how electric charges and currents create electric and magnetic fields, and how they interact.
- Gauss's Law for Electricity: Electric charges produce an electric field.
- Gauss's Law for Magnetism: There are no magnetic monopoles; magnetic field lines are always closed loops.
- Faraday's Law of Induction: A changing magnetic field creates an electric field (the basis for generators).
- Ampere-Maxwell Law: Electric currents and changing electric fields create magnetic fields.

Modern Technology Applications

- Wireless Communication (Wi-Fi, 5G, Phones):
- Information is transmitted through the air as electromagnetic waves. Antennas convert electrical signals into waves and vice-versa, based on the interrelation of electric and magnetic fields.
- Wireless Charging:Uses Induction. A charging pad creates a changing magnetic field that induces an electric current in your phone's coil to charge the battery

- Microwave Ovens:
- The oscillating electric field of the microwaves forces water molecules in food to rotate rapidly, generating heat through friction to cook the food.
- Electric Motors & Generators:
- Generators convert mechanical motion into electricity (Faraday's Law), while motors convert electricity into motion using magnetic forces.
- MRI Machines:
- Use powerful magnetic fields and radio waves to create detailed images of the body's internal structures without harmful radiation.

The first equation

- The first equation relates electric charge closed within a closed surface to the surrounding electric field. It describes with mathematical clarity how the divergence of an electric field is affected by charges. It states that net outward flux of electric displacement vector through closed surface is equal to the net charge enclosed by the surface but it is not related to the shape and size of that surface.

$$\nabla . E = \rho / \varepsilon 0$$

The second equation:

- The second equation states that the magnetic flux through closed surface is zero. This is due to real world magnetic charges coming in pairs (referred to as magnetic dipoles), with two charges giving rise to opposite magnetic field divergences which cancel each other out. Gauss's law for magnetism is also mathematical form of the assertion that single magnetic charges referred to as magnetic monopoles, do not exists in our physical world.

$$\nabla . B = 0$$

The third equation

- The third equation describes how a time varying magnetic field can create an electric field and acts as the source of induced e.m.f. This is the operating principle behind many electric generators like hydro-electric generator that drives electricity through the power grid.

$$\nabla \times E = -\partial B / \partial t$$

The fourth equation

- Ampere's law with Maxwell's correction, states that magnetic fields can be regenerated in two ways:

by electric current and by changing electric fields

- The idea that a magnetic field can be induced by changing electric field follows from the modern concept of displacement current which was introduced to maintain the solenoidal nature of Ampere's law in vacuum capacitor circuit. Maxwell's current applies to polarization current in a dielectric medium, and it sits adjacent to the modern displacement current in Ampere's law.

$$\nabla \times B = \mu_0(J + E_o \frac{\partial E}{dt})$$